

**Semester 1 Examination 2012**

**Question/Answer Booklet**

**MATHEMATICS 3AB**

**Section Two:**

**Calculator-assumed**

Name of Student: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_Marking key\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Time allowed for this section**

Reading time before commencing work: 10 minutes

Working time for this section: 100 minutes

**Materials required/recommended for this section**

***To be provided by the supervisor***

This Question/Answer Booklet

Formula Sheet (retained from Section One)

***To be provided by the student***

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler,

Highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,

and up to three calculators satisfying the conditions set by the Curriculum

Council for this examination

**Important note to students**

No other items may be used in this section of the examination. It is **your** responsibility to ensure

that you do not have any unauthorised notes or other items in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**Structure of this paper**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time  (minutes) | Marks available | Percentage of exam |
| Section One  Calculator-free | 7 | 7 | 50 | 50 |  |
| Section Two  Calculator-assumed | 13 | 13 | 100 | 100 |  |

|  |  |  |
| --- | --- | --- |
| Total | 150 | 100 |

**Instructions to students**

1 Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer. If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued. i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

2 **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

3 It is recommended that you **do not use pencil**, except in diagrams.

**Section Two: Calculator-assumed (100 marks)**

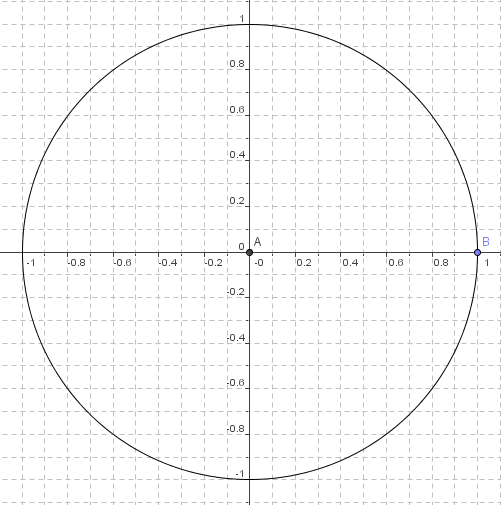
This section has **thirteen (13)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 100 minutes

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Question 8 (5 marks)**

Use the Unit circle below (there must be evidence of using the unit circle before full marks will be awarded)



(i) to evaluate the value of  (2)

|  |
| --- |
| **Solution** |
| From unit circle, cos 1200 = -0.5  Therefore = 2 x (-0.5) = -1 |
| **Specific behaviours** |
| ✓ value for 1200  ✓ correct value for |

(ii) to find the value(s) of *x* , to the nearest whole number, such that

 (3)

|  |
| --- |
| **Solution** |
| Read the two angles such that the y value is 0.9, giving 660 and 1140  DIvide both angles by 2 to give x = 330 and 570 |
| **Specific behaviours** |
| ✓ two values for 2x  ✓✓ two angles for x |

**Question 9 (9 marks)**

Consider the following sequence of numbers 2, 5, 2, 9, 2, 13, 2, 17, …

1. Write down the next two terms of the sequence, if the pattern continues. (1)

|  |
| --- |
| **Solution** |
| 2,21 |
| **Specific behaviours** |
| ✓ or X |

1. In this sequence, T1=T3=T5= etc, complete the sentence,

“ If n is an odd number, then

Tn= \_\_ , T1 = “ (1)

|  |
| --- |
| **Solution** |
| Tn = 2, T1 = 2 |
| **Specific behaviours** |
| ✓ or X |

1. Write a recursive formula for the sequence 5, 9, 13, 17 ,… (2)

|  |
| --- |
| **Solution** |
| Tn+1 = Tn + 4, T1 = 5 |
| **Specific behaviours** |
| ✓✓ or X |

1. Write down the values of T99 and T100. (2)

|  |
| --- |
| **Solution** |
| T99 = 2, T100 = 201 |
| **Specific behaviours** |
| ✓✓ or X |

1. Which term is 61? (1)

|  |
| --- |
| **Solution** |
| From CAS, T30 is 61 |
| **Specific behaviours** |
| ✓ or X |

1. Calculate the sum of the first 100 terms of the sequence. (2)

|  |
| --- |
| **Solution** |
| 5 + 7 + 9 + … + 201 = 5150  Sum of the first 100 terms = 5150 + 100 = 5250 |
| **Specific behaviours** |
| ✓sum of the fifty 2’s and sum of 5,7,9,…201  ✓ sum of the first 100 terms |

**Question 10 (6 Marks)**

Four digit numbers may be formed using the digits 0, 2, 4, 6, 8 such that no digit can be used more than once. How many of these 4- digit numbers

(i) may be formed? (1)

|  |
| --- |
| **Solution** |
| 5! = 120 |
| **Specific behaviours** |
| ✓ or X |

(ii) begin with 8? (1)

|  |
| --- |
| **Solution** |
| 1x4x3x2 = 24 |
| **Specific behaviours** |
| ✓ or X |

(iii) end in a 2 or a 4? (1)

|  |
| --- |
| **Solution** |
| 1x4x3x2 x2 = 48 |
| **Specific behaviours** |
| ✓ or X |

(iv) are less than 486? (2)

|  |
| --- |
| **Solution** |
| 400’s: 5  200’s: 6  Therefore total is 11 |
| **Specific behaviours** |
| ✓✓ |

(v) What is the probability that a randomly picked 4-digit number from this set is more than 8640? (1)

|  |
| --- |
| **Solution** |
| Only 8642 |
| **Specific behaviours** |
| ✓ or X |

**Question 11 (6 marks)**

(a) The distribution of heights of a plant is normally distributed with a mean of 72 cm and a standard deviation of 6 cm. If there were 100 plants, estimate the number of plants which are less than 64cm in height. (2)

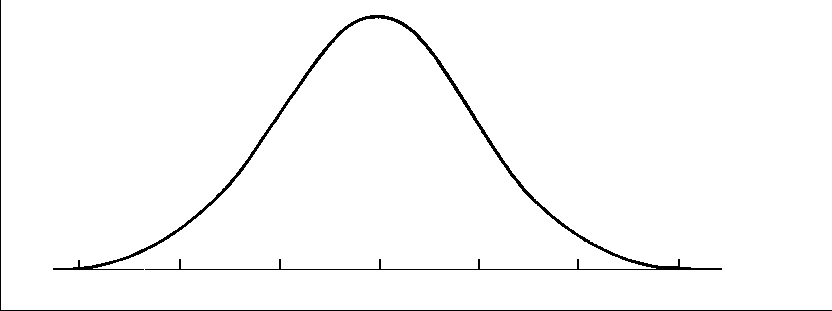
|  |
| --- |
| **Solution** |
| P(X<64) = 0.0912  Number of plants = 0.0912 x 100 = 9 |
| **Specific behaviours** |
| ✓ probability  ✓ Number |

(b) Achievement awards in a national Mathematics competition are allocated as

* High Distinctions to the top 15% of performances
* Distinctions to the next 25%
* Credits to the next 30%
* Participations to the remainder

If the scores were normally distributed with a mean of 75 and a standard deviation of 12, find the minimum score a competitor has to achieve to be awarded a

0.1



HD

D

C

0.2

0.25

0.15

1. Credit (2)

|  |
| --- |
| **Solution** |
| P(X<Credit) = 0.3  C = 68.7  Minimum score for a Credit is 68.7 |
| **Specific behaviours** |
| ✓ correct probability  ✓ minimum score for Credit |

1. Distinction (1)

|  |
| --- |
| **Solution** |
| P(X>D) = 0.40  D = 78  Minimum score for Distinction is 78 |
| **Specific behaviours** |
| ✓ minimum score for a Distinction |

1. High Distinction (1)

|  |
| --- |
| **Solution** |
| P(X>HD) = 0.15  HD = 87  Minimum score for a High Distinction is 87 |
| **Specific behaviours** |
| ✓ or X |

**Question 12 (6 marks)**

Given the triangle below,



a2

300

(i) Find a simplified algebraic expression for the area of the triangle, given that . (2)

|  |
| --- |
| **Solution** |
| Area =  = |
| **Specific behaviours** |
| ✓ area of non -right triangle formula with the correct sides  ✓ correct expression |

(ii) If the area is 17.5 units2, determine the value of ‘a’. (2)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ equates area expression to 17.5  ✓ solves correctly for “a” |

(iii) Hence, state the lengths of the triangle. (2)

|  |
| --- |
| **Solution** |
| Lengths are 7, 5, 10 |
| **Specific behaviours** |
| ✓ substitutes a = 3 into each of the three sides  ✓ three correct lengths |

**Question 13 (8 marks)**

(a) A group of fifteen students travelling to Tasmania have their bags weighed at the airport

check-in counter. The weights of the bags in kilograms are listed in order as follows;

m, 9, n, 10, 15, 18, 21, p, 25, 29, 35, q, 38, r, 43.

The following statistics of this set of data are known.

=24, Median = 21, Interquartile range = 27.

There are two modes of which 10 is one of them.

The minimum score is 2 less than the smaller mode.

Determine the values of m, n, p, q, r. (5)

|  |
| --- |
| **Solution** |
| p = 21  q = 37  n = 10  m = 8  r = 41 |
| **Specific behaviours** |
| ✓✓✓✓✓ 1 mark each |

**Question 13 (continued)**

(b) In collecting data, biased samples and poor methods of collection can lead to invalid inferences. Consider the following method to determine the extent of unemployment in a local community.

“A team of data collectors phoned five randomly selected households on every twentieth page of the local telephone directory during regular office hours”.

1. List two ways which might suggest this data collection method cause bias? (2)

|  |
| --- |
| **Solution** |
| * during office hours, some unemployed people will be contacted but so too will home-carers, retired individuals, people running a business from home, etc * not all people are listed on the telephone directory * Thus those interviewed will not provide the right kind of data to make supportable inferences |
| **Specific behaviours** |
| ✓✓ 2 valid reasons |

1. Suggest how this bias could be avoided. (1)

|  |
| --- |
| **Solution** |
| * Door knock after regular working hours * Ring after regular working hours to enable more people to be reached * Survey form |
| **Specific behaviours** |
| ✓ any 1 valid suggestion |

**Question 14 (10 marks)**

(a) The graph of *f(x)* is dilated parallel to the *x-*axis by a scale factor of 2, followed by a translation of 4 units in the positive direction of the *x-*axis. What is the equation of the final graph obtained? (2)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓✓  and -4 OR ✓✓  and -2 |

(b) The function *y=x3* is transformed to *y = -2( x +5 )3*  . Describe the transformation in order. (3)

|  |
| --- |
| **Solution** |
| Translated 5 units to the left, followed by a dilation parallel to the y-axis by a scale factor of 2 and a reflection in the x-axis |
| **Specific behaviours** |
| ✓ transformation in correct order  ✓✓ translation 5 left, dilation 2 parallel to y axis and reflection in the x-axis |

(c) The graph of *y=f(x)* is given below



A

1. Sketch the graph of *y=3 - f(x)* on the axes below. (2)



|  |
| --- |
| **Solution** |
| As shown above |
| **Specific behaviours** |
| ✓ reflected shape  ✓ all correct points (0,3), (1,2), (4,-2) |

**Question 14 (continued)**

1. Write the coordinates of A if *y = - f(-x+3).*  (2)

|  |
| --- |
| **Solution** |
| Coordinates of A = (2, -1) |
| **Specific behaviours** |
| ✓ working  ✓ correct coordinates of A |

(d) Describe how the graph of *f(x)* can be used to obtain the graph of *g(x).* (1)



f(x)

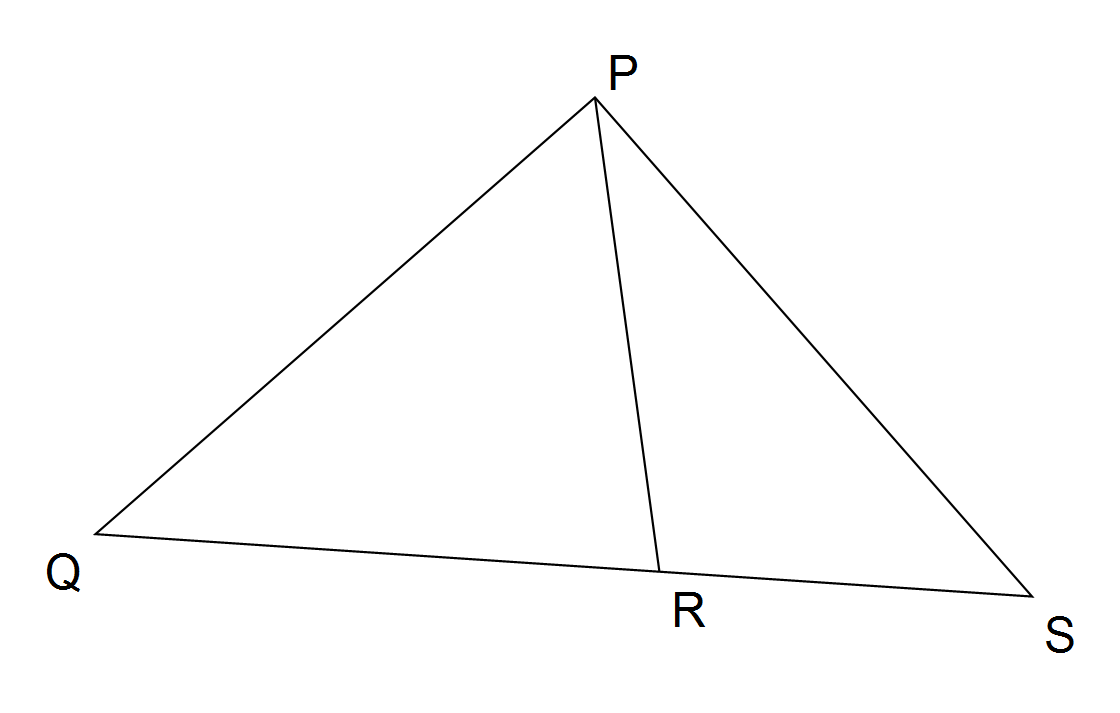
g(x)

|  |
| --- |
| **Solution** |
| Reflection about the y-axis  Or f(x) is translated 2 units left |
| **Specific behaviours** |
| ✓ or X |

**Question 15 (7 marks)**

1. In the given diagram, PR = 8 cm, QR = 5 cm and the sizes of angles PRQ and PSR are

and 40 respectively. Calculate



1. the exact length of PQ

(2)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ Correct use of Cosine rule  ✓ Correct value for PQ |

1. the length of PS correct to 1 decimal place (2)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct use of sine rule  ✓ correct answer to 1 decimal place |

1. Triangle ABC is a non-right angled triangle. Given angle A = 30and b = 9. (3)

Find the value(s) of angle B.



(2)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct use of sine rule  ✓✓correct answers |

**Question 16 (8 marks)**

(a) The following table shows statistics of the results of the five mathematics tests for John, Ryan, Julie and Gary.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Mean | Standard deviation | Range |
| John | 13.6 | 3.8 | 7 |
| Ryan | 13.9 | 4.2 | 7 |
| Julie | 13.8 | 2.7 | 7 |
| Gary | 14.0 | 3.4 | 7 |

Which student has the most consistent results? State a reason why you chose this student. (2)

|  |
| --- |
| **Solution** |
| Julie because she has the smallest spread of marks (standard deviation) |
| **Specific behaviours** |
| ✓ Julie  ✓ lowest standard deviation |

(b) Mrs Kendall decided to survey a sample of 10% of the students at her school. The school enrolment is shown in the table below.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Year | 7 | 8 | 9 | 10 | 11 | 12 | Total |
| Number of students | 225 | 232 | 233 | 230 | 150 | 130 | 1200 |

She surveyed the same number of students in each year group. Explain how would the numbers of students surveyed in Year 10 and Year 11 have changed if she had chosen to use a stratified sample based on year groups? (3)

|  |
| --- |
| **Solution** |
| Same number for each group  i.e.  ✓  Year 10: 10% of 230 = 23  increase of 3 ✓  Year 11: 10% of 150 = 15  decrease of 5 ✓ |
| **Specific behaviours** |
| As indicated above |

**Question 16 (continued)**

(c) Solve the indicial equation showing all working steps (3)



|  |
| --- |
| **Solution** |
| Equating indices: |
| **Specific behaviours** |
| ✓ expresses as powers of 3  ✓ equates indices  ✓ solves correctly for x |

**Question 17 (9 marks)**

Josh bought a second hand Hyundai for $4500. He borrowed the full sum of $4500 from NO Frills Finance Company and agreed to make a minimum payment of $300 a month. For a year he pays the minimum monthly payment. At the end of the year, he checks his statement from the Finance company and is surprised that he still owes $1683.50. He created a spreadsheet to check whether the amount owing is correct.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Month | Previous Balance | Interest | Balance Due | Payment | Balance after payment |
| 1 | 4500 | 90 | 4590 | 300 | 4290 |
| 2 | 4290 | 85.80 | 4375.80 | 300 | 4075.80 |
| 3 | 4075.80 | 81.50 | 4157.30 | 300 | 3857.30 |
| 4 | 3857.30 | 77.10 | 3934.50 | 300 | 3634.50 |
| 5 | 3634.50 | 72.70 | 3707.20 | 300 | 3407.20 |
| 6 | 3407.20 | 68.10 | 3475.30 | 300 | 3175.30 |
| 7 | 3175.30 | 63.50 | 3238.80 | 300 | 2938.80 |
| 8 | 2938.80 | 58.80 | 2997.60 | 300 | 2697.60 |
| 9 | 2697.60 | 53.90 | 2751.50 | 300 | 2451.50 |
| 10 | 2451.50 | 49.10 | 2500.60 | 300 | 2200.60 |
| 11 | 2200.60 | 44.00 | 2244.60 | 300 | 1944.60 |
| 12 | 1944.60 | 38.90 | 1983.50 | 300 | 1683.50 |

Note: The values in this table have been rounded to the nearest ten cents.

(i) Calculate the annual rate of interest Josh is paying. (1)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ or X |

(ii) If Josh continues to pay the monthly payment of $300, it will take him a total of 19 months to pay off his car. What would be the total amount of his payments? (2)

|  |
| --- |
| **Solution** |
| Total amount of his payments = |
| **Specific behaviours** |
| 🗸✓ calculation and correct answer |

(iii) Josh decides to reduce his social life activities in order to pay an extra $200 a month from the 13th payment onwards. Values have been rounded to the nearest ten cents.

Complete the table of calculation below. (2)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Month | Previous Balance | Interest | Balance Due | Payment | Balance after payment |
| 13 | 1683.50 | **33.70** | 1717.20 | 500 | 1217.20 |
| 14 | 1217.20 | 24.30 | **1241.50** | 500 | 741.50 |
| 15 | 741.50 | 14.80 | 756.30 | 500 | **256.30** |

|  |
| --- |
| **Solution** |
| As shown in table |
| **Specific behaviours** |
| ✓✓ for the three correct values |

**Question 17 (continued)**

(iv) How long does it take Josh to repay the loan? (1)

|  |
| --- |
| **Solution** |
| 16 months |
| **Specific behaviours** |
| ✓ or X |

(v) What is the total amount that Josh will pay over the period with the increased repayment ? (2)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓✓ calculation and correct answer |

(vi) How much money does Josh save by paying in this way? (1)

|  |
| --- |
| **Solution** |
| $ 5 403.50 - $ 5 361.40 = $ 42.10 |
| **Specific behaviours** |
| ✓ or X |

**Question 18 (8 marks)**

The final result of the students in Mrs Bond’s classes of 3CDMAT and 3CDMAS are summarised in the frequency table below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Mark in percent | 3CDMAT | Cumulative frequency | 3CDMAS | Cumulative frequency |
| 0 – 19 | 2 | 2 | 1 | 1 |
| 20 – 39 | 2 | 4 | 3 | 4 |
| 40 – 59 | 5 | C | A | 7 |
| 60 – 79 | D | 20 | 9 | B |
| 80 - 99 | 7 | 27 | 5 | 21 |

(a) Find the values of A, B C and D (2)

|  |
| --- |
| **Solution** |
| A = 3, B = 16, C = 9, D = 11 |
| **Specific behaviours** |
| ✓✓ for all four correct answers |

(b) Calculate the mean and standard deviation of the 3CDMAS marks (2)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓✓ 1 mark each |

(c) If a student from one of these classes is picked at random, what is the probability that the

student

1. received a mark greater than or equal to 60 ? (1)
2. was in the 3CDMAT class? (1)
3. was in the 3CD MAS class and received a mark  80 ? (1)
4. was in the 3CDMAS class if the student received a mark

between 60 and 79 inclusive ? (1)

|  |
| --- |
| **Solution** |
| 1. (ii)  (iii)  (iv) |
| **Specific behaviours** |
| ✓✓✓✓ 1 mark each |

**Question 19 (9 marks)**

1. A and B are events such that 
2. Represent the diagram in the Venn diagram below. (3)



|  |
| --- |
| **Solution** |
| As shown in diagram |
| **Specific behaviours** |
| ✓ 0.65  ✓ 0.05  ✓ 0.1 |

1. Hence, find P(A) (1)

|  |
| --- |
| **Solution** |
| 0.85 |
| **Specific behaviours** |
| ✓ or X |

1. Hence find  (1)

|  |
| --- |
| **Solution** |
| 0.75 |
| **Specific behaviours** |
| ✓ or X |

**Question 19 (continued)**

1. The test results for a group of 45 students involved in a reading program are displayed below in the form of a pair of box plots. The data collected shows the reading speed of the students both before and after the administration of the program.



NOTE : IN THE EXAM the before and After was swapped around accidently.

For each of the following, answer True (T) or False (F):- (4)

(i) For the “after “ data, the median has increased by 8 and the interquartile range has increased by 4.

(ii) Both distributions have the same median.

(iii) The ‘after’ set of scores is more evenly distributed than the ‘before’ set of scores.

(iv) The range of both sets of scores is the same.

|  |
| --- |
| **Solution** |
| 1. F 2. F 3. F 4. F |
| **Specific behaviours** |
| ✓✓✓✓ 1 mark each |

**Question 20 (9 marks)**

The point P(1,*k*) lies on the curve with equation 

(i) Find the value of *k.* (1)

|  |
| --- |
| **Solution** |
| k= 8 |
| **Specific behaviours** |
| ✓ or X |

(ii) Express in a completely factorised form. (2)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expand  ✓ factorised form |

(iii) On the axes below sketch the curves with the following equations

 and , showing clearly where they meet the axes. (3)



|  |
| --- |
| **Solution** |
| As displayed in diagram |
| **Specific behaviours** |
| ✓✓ 1 mark each for f(x) and g(x)  ✓ shows clearly x and y intercepts, turning points |

**Question 20 (continued)**

(iv) Read from the graph the **number** of solutions to the equation. (1)



|  |
| --- |
| **Solution** |
| 4 solutions |
| **Specific behaviours** |
| ✓ or X |

(v) By adding **one** suitable function on the axes, show how you solve and estimate the solution(s) from your graph, correct to 1 decimal place. (2)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ draws the line y=x as shown on the graph  ✓ reads the x values where y=x intersects with |